Einstein, Kant, and the Relativized *A Priori*

Michael Friedman

Abstract I argue that Einstein's creation of both special and general relativity instantiates Reichenbach's conception of the relativized a priori. I do this by showing how the original Kantian conception actually contributes to the development of Einstein's theories through the intervening philosophical and scientific work of Helmholtz, Mach, and Poincaré.

Kant's original version of transcendental philosophy took both Euclidean geometry and the Newtonian laws of motion to be synthetic a priori constitutive principles –which, from Kant's point of view, function as necessary presuppositions for applying our fundamental concepts of space, time, matter, and motion to our sensible experience of the natural world.[1] Although Kant had very good reasons to view the principles in question as having such a constitutively a priori role, we now know, in the wake of Einstein's work, that they are not in fact a priori in the stronger sense of being fixed necessary conditions for all human experience in general, eternally valid once and for all. And it is for precisely this reason that Kant's original version of transcendental philosophy must now be radically reconceived.

It was Hans Reichenbach, in *Relativitätstheorie und Erkenntnis Apriori* (1920), who first proposed the idea of *relativizing* Kantian constitutively a priori principles of geometry and mechanics. Such principles still function, throughout the development from Newton to Einstein, as necessary presuppositions for applying our (changing) conceptions of space, time, and motion to our sensible experience, but they are no longer eternally valid once and for all. Instead of global necessary conditions for all human experience in general, we have merely local necessary conditions for the empirical application of a particular mathematical–physical theory at a given time and in a given historical context. For example, while Euclidean geometry and the

M. Friedman
Stanford University

[1] For details on Kant's understanding of Euclidean geometry and the fundamental principles of Newtonian mechanics see Friedman (1992).

M. Bitbol et al. (eds.), *Constituting Objectivity*,

Newtonian laws of motion are indeed necessary conditions for giving empirical meaning to the Newtonian theory of universal gravitation, the situation in Einstein's general theory relativity is quite different. The crucial mediating role between abstract mathematical theory and concrete sensible experience is now played by the light principle and the principle of equivalence, which together insure that Einstein's revolutionary new description of gravitation by a four-dimensional geometry of variable curvature in fact says something about concrete empirical phenomena: namely, the behavior of light and gravitationally interacting bodies.

In my recent book, *Dynamics of Reason* (2001), I have taken up, and further developed, Reichenbach's idea. But my implementation of this idea of relativized constitutively a priori principles (of geometry and mechanics) essentially depends on an historical argument describing the developmental process by which the transition from Newton to Einstein actually took place, as mediated, in my view, by the parallel developments in scientific philosophy involving, especially, Hermann von Helmholtz, Ernst Mach, and Henri Poincaré. However, since this argument depends on the concrete details of the actual historical process in question, it would therefore appear to be entirely contingent. How, then, can it possibly be comprehended within a properly *transcendental* philosophy? Indeed, once we have given up on Kant's original ambition to delineate in advance the a priori structure of all possible scientific theories, it might easily seem that a properly transcendental argument is impossible. We have no way of anticipating a priori the specific constitutive principles of future theories, and so all we can do, it appears, is wait for the historical process to show us what emerges a posteriori as a matter of fact. So how, more generally, can we develop a philosophical understanding of the evolution of modern science that is at once genuinely historical and properly transcendental?[2]

Let us begin by asking how Kant's original transcendental method is supposed to explain the sense in which certain fundamental principles of geometry and mechanics are, in fact, both a priori and necessary. This method, of course, appeals to Kant's conception of the two rational faculties of sensibility and understanding. The answer to the question "how is pure mathematics possible?" appeals to the necessary structure of our pure sensibility, as articulated in the Transcendental Aesthetic of the *Critique of Pure Reason*; the answer to the question "how is pure natural science possible?" appeals to the necessary structure of our pure understanding, as articulated in the Transcendental Analytic. Yet there is an obvious objection to this procedure: how can such proposed transcendental explanations inherit the (assumed) a priori necessity of the sciences whose possibility they purport to explain unless we can also somehow establish that they are the *unique* such explanations?[3] From our present point of view, for example, it does not appear that Kant's explanation of the possibility

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[2] I am especially indebted to Charles Parsons for raising this problem of historical contingency and stimulating me to take it very seriously.

[3] Kant often makes such claims to explanatory uniqueness, for example, in the Transcendental Exposition of the Concept of Space added to the second ["B"] edition (B 41): "Therefore, only our explanation makes the *possibility of geometry* as an a priori synthetic cognition comprehensible. Any mode of explanation that does not achieve this, even if it appeared to be similar to ours, can be most securely distinguished from ours by this criterion." I am indebted to Dagfinn Føllesdal for emphasizing to me the importance of the problem of uniqueness in this connection.

of pure mathematics is uniquely singled out in any way; on the contrary, our greatly expanded conception of purely logical or analytic truth suggests that an appeal to the faculty of pure sensibility may, after all, be entirely superfluous. Indeed, from the point of view of the anti-psychological approach to such questions that dominated much of twentieth-century analytic philosophy, it appears that all consideration of our subjective cognitive faculties is similarly explanatorily superfluous.

In Kant's own intellectual context, however, explanations of scientific knowledge in terms of our cognitive faculties were the norm – for empiricists, rationalists, and (of course) Aristotelians. Everyone agreed, in addition, that the relevant faculties to consider were the senses and the intellect; what was then controversial was the precise nature and relative importance of the two. Empiricist views, which denied the existence of the pure intellect or its importance for scientific knowledge, were, for Kant, simply out of the question, since they make a priori rational knowledge incomprehensible.[4] Moreover, the conception of the pure intellect that was most salient for Kant was that of Leibniz, where the fundamental structure of this faculty is delineated, in effect, by the logical forms of traditional Aristotelian syllogistic. But this conception of the pure intellect, Kant rightly saw, is entirely inadequate for representing, say, the assumed infinite extendibility and divisibility of geometrical space, which had recently proven itself to be both indispensable and extremely fruitful in Newtonian mathematical physics.[5] Nevertheless, Newton's own conception of space as the divine sensorium was also entirely unacceptable on theological and metaphysical grounds, and so the only live alternative left to Kant was the one he actually came up with: space is a pure form of our sensibility (as opposed to the divine sensibility), wherein *both* (infinitely iterable) geometrical construction *and* the perception of spatial objects in nature (like the heavenly bodies) then become first possible.[6]

Kant's answers to the questions "how is pure mathematics possible?" and "how is pure natural science possible" therefore operate against the background of an

[4] Thus, in considering the questions "how is pure mathematics possible?" and "how is pure natural science possible?" in section VI of the Introduction to the second edition of the *Critique*, Kant simply takes it for granted that the actual existence of these sciences puts the existence of synthetic a priori knowledge entirely beyond all doubt. In particular, in considering Hume's skepticism concerning the necessity of the causal relation – which then leads to skepticism about the possibility of any a priori metaphysics – Kant blames this result on Hume's insufficiently general understanding of the problem (B 20): "[H]ume would never have arrived at this assertion, which destroys all pure philosophy, if he had kept our problem before his eyes in its [full] generality; for he would then have seen that, according to his argument, there could also be no pure mathematics (for it certainly contains synthetic a priori propositions), and his good sense would then surely have saved him from this assertion." Similarly, while considering (in section 14 of the second edition) the circumstance that neither Locke nor Hume posed the problem of the transcendental deduction, and instead attempted a psychological or empirical derivation of the pure concepts of the understanding, Kant concludes (B 127–128): "But the *empirical* derivation which both fell upon cannot be reconciled with the actuality of the a priori scientific cognition that we have—namely of *pure mathematics* and *universal natural science*—and is thus refuted by this fact [*Faktum*]."

[5] Again, see Friedman (1992, chapters 1 and 2) for details.

[6] See my "Newton and Kant on Absolute Space: From Theology to Transcendental Philosophy" (this volume) for details.

existing set of intellectual resources in a particular historical context. Geometry, for Kant, is limited to the classical system of Euclid; the pure understanding or pure intellect is delimited by the logical forms of Aristotle; the available conceptions of space and time are exhausted by the Leibnizean and Newtonian alternatives; and so on. Kant's theory of our faculties of sensibility and understanding can only be understood against the background of precisely these resources – mathematical, logical, metaphysical, and theological – as Kant delicately navigates within them and eventually radically transforms them. The revolutionary and completely unexpected result, that space and time are pure forms of our (human) faculty of sensibility and that, considered independently of sensibility, our faculty of understanding yields no (theoretical) cognition at all, then emerges as the practically unique solution to the problem set by the existing intellectual resources: it is the only available conception of our rational faculties that does simultaneous justice to both Newtonian mathematical physics and Leibnizean (as opposed to Newtonian) natural theology and metaphysics.

It is of course entirely contingent that Kant operated against the background of precisely these intellectual resources, just as it is entirely contingent that Kant was born in 1724 and died in 1804. Given these resources, however, and given the problems with which Kant was faced, the solution he came up with is not contingent. On the contrary, the intellectual situation in which he found himself had a definite "inner logic" –mathematical, logical, metaphysical, and theological – which allowed him to triangulate, as it were, on a practically unique (and in this sense necessary) solution.

Beginning with this understanding of Kant's transcendental method and its associated rational necessity, we can then see a way forward for extending this method to post-Kantian developments in both the mathematical exact sciences and transcendental philosophy. We can trace out how the "inner logic" of the relevant intellectual situation evolves and changes after Kant in response to both new developments in the mathematical exact sciences themselves and the manifold and intricate ways in which post-Kantian scientific philosophers attempted to reconfigure Kant's original version of transcendental philosophy in light of these developments. That each of these successive new intellectual situations has its own "inner logic" implies that the enterprise does not collapse into total contingency; that, in addition, they successively evolve out of, and in light of, Kant's original system suggests that it may still count as transcendental philosophy. In my reconceived version of transcendental philosophy, therefore, integrated intellectual history of both the exact sciences and scientific philosophy takes over the role of Kant's original transcendental faculty psychology.

Hermann von Helmholtz's neo-Kantian scientific epistemology, for example, had deep roots in Kant's original conception. In particular, Helmholtz developed a distinctive conception of space as a "subjective" and "*necessary* form of our external intuition" in the sense of Kant; and, while this conception was certainly developed within Helmholtz's *empirical* program in sensory psychology and psycho-physics, it nevertheless retained important "transcendental" elements.[7] More specifically,

[7] For Helmholtz's characteristic combination of empirical (or "naturalistic") and transcendental (or "normative") elements see Hatfield (1990). For my reading of Helmholtz's conception of space and geometry see Friedman (1997, 2000).

space is "transcendental," for Helmholtz, in so far as the principle of free mobility (which allows arbitrary continuous motions of rigid bodies) is a necessary condition for the possibility of spatial measurement – and, indeed, for the very existence of space and spatial objects. Moreover, the condition of free mobility represents a natural generalization of Kant's original (Euclidean) conception of geometrical construction, in the sense that Euclidean constructions with straight-edge and compass, carried out within Kant's form of spatial intuition, are generated by the group of specifically Euclidean rigid motions (translations and rotations). The essential point, however, is that free mobility also holds for the classical non-Euclidean geometries of constant curvature (hyperbolic and elliptic), and so it is no longer a "transcendental" and "necessary" condition of our spatial intuition, for Helmholtz, that the space constructed from our perception of bodily motion obeys the specific laws of Euclidean geometry. Nevertheless, Helmholtz's generalization of the Kantian conception of spatial intuition is, in an important sense, the *minimal* (and in this sense unique) such generalization consistent with the nineteenth-century discovery of non-Euclidean geometries.[8]

The great French mathematician Henri Poincaré then transformed Helmholtz's conception in turn. In particular, Poincaré's use of the principle of free mobility (which plays a central role in his philosophy of geometry) is explicitly framed by a hierarchical conception of the mathematical sciences, beginning with arithmetic and proceeding through analysis, geometry, mechanics, and empirical physics; and, for Poincaré, it follows that one should thereby explain the application of pure mathematics to our perceptual experience precisely in terms of the hierarchy in question.[9]

Poincaré, to begin with, views pure arithmetic as a synthetic a priori science involving the ineliminable use of an essentially non-logical principle of reasoning by recurrence or mathematical induction. This principle, for Poincaré, rests on the fundamental intuition of indefinitely repeatable succession or iteration – a conception which is very close indeed to Kant's original philosophy of arithmetic.[10] At the next lower level of the hierarchy is analysis or the theory of mathematical magnitude (also explained with an eye towards its intuitive meaning and perceptual application); and, at a crucial intermediate level, below the sciences of arithmetic and analysis but above the sciences of mechanics and empirical physics, is the science of geometry. In particular, whereas the mathematical structure and empirical

[8] Bernhard Riemann's general theory of manifolds includes spaces of *variable* curvature not satisfying the condition of free mobility, and it is for precisely this reason that Hermann Weyl later attempted to generalize Helmholtz's approach to comprehend the (four-dimensional) (semi-) Riemannian geometries of variable curvature used in Einstein's general theory of relativity. Moreover, as I explain in Friedman (2000, pp. 209–211), Weyl, too, conceived his work as a generalization of Kant's original theory of space as an (a priori) "*form of experience*." The important point here, however, is that Helmholtz is "closer" to Kant's original theory (in so far as his generalization preserves the possibility of geometrical constructions analogous to Euclid's), whereas Weyl's work arises only as a further generalization, in turn, of Helmholtz's.

[9] This hierarchical conception is developed especially clearly in *La Science et l'Hypothèse* (1902). For details see Friedman (1999, chapter 4, 2000).

[10] For Poincaré's philosophy of arithmetic, in particular, see Folina (1992).

meaning of the science of geometry presupposes the existence of the two preceding sciences, it is presupposed, in turn, by the two succeeding ones.

This hierarchical conception of the mathematical sciences underlies Poincaré's fundamental disagreement with Helmholtz. For Helmholtz, as we have seen, the principle of free mobility expresses the necessary structure of our form of external intuition, and, following Kant, Helmholtz views all empirical investigation as necessarily taking place within this already given form. Helmholtz's conception is Kantian, that is, in so far as space has a "necessary form" expressed in the condition of free mobility, but it is also empiricist in so far as which of the three possible geometries of constant curvature obtains is then determined by experience. For Poincaré, by contrast, although the principle of free mobility is still fundamental, our actual perceptual experience of bodily "displacements" arising in accordance with this principle is far too imprecise and indefinite to yield the empirical determination of a specific mathematical geometry: our only option, at this point, is to *stipulate* Euclidean geometry by convention, as the simplest and most convenient idealization of our actual perceptual experience. In particular, experiments with putatively rigid bodies, for Poincaré, involve essentially physical processes at the level of mechanics and experimental physics, and these sciences, in turn, *presuppose* that the science of geometry is already firmly in place. In the context of Poincaré's hierarchy, therefore, the principle of free mobility expresses our necessary freedom to choose – by a "convention or definition in disguise" – which of the three classical geometries of constant curvature is the most suitable idealization of physical space.

One of the most important applications of Poincaré's hierarchical conception involves his characteristic perspective on the problem of absolute space and the relativity of motion explained in his discussion of the next lower level in the hierarchy: (classical) mechanics. Poincaré's key idea is that what he calls the (physical) "law of relativity" rests squarely on the "relativity and passivity of space" and therefore reflects the circumstance, essential to free mobility, that the space constructed from our experience of bodily displacements is both homogeneous and isotropic: all points in space, and all directions through any given point, are, necessarily, geometrically equivalent.[11] Thus, Poincaré's conception of the relativity of motion depends

[11] The "law of relativity" is first introduced in chapter V, "Experience and Geometry," of *La Science et l'Hypothèse* (1902, p. 96, 1913b, p. 83): "The laws of the phenomena which will happen [in a material system of bodies] will depend on the state of these bodies and their mutual distances; but, because of the relativity and passivity of space, they will not depend on the absolute position and orientation of this system. In other words, the state of the bodies and their mutual distances will depend only on the state of the same bodies and their mutual distances at the initial instant, but they will not depend at all on the absolute initial position of the system and its absolute initial orientation. This is what I shall call, for the sake of brevity, *the law of relativity*." Moreover, "in order fully to satisfy the mind," Poincaré continues, the phenomena in question should also be entirely independent of "the velocities of translation and rotation of the system, that is to say, the velocities with which its absolute position and orientation vary" (1902, p. 98, 1913b, p. 85). Thus, because of "the relativity and passivity of space," the absolute position or orientation of a system of bodies in space can have no physical effect whatsoever, and neither can any *change* (velocity) of such absolute position or orientation. In emphasizing that Poincaré's treatment of the relativity of motion rests squarely on his philosophy of space and geometry, I am in very substantial agreement with the excellent discussion in DiSalle (2006, section 3.7).

entirely on his philosophy of geometry, and this is especially significant, from our present point of view, because Poincaré's ideas on the relativity of motion were also inextricably entangled with the deep problems then afflicting the electrodynamics of moving bodies that were eventually solved (according to our current understanding) by Einstein's special theory of relativity.

I shall return to Einstein below, but I first want to emphasize that the connection Poincaré makes between his philosophy of geometry and the relativity of motion represents a continuation of a problematic originally prominent in Kant. Helmholtz, as we have seen, transformed Kant's philosophy of space and geometry, and Ernst Mach, among others, participated in a parallel transformation of Kant's approach to the relativity of motion – which finally eventuated in the modern concept of an inertial frame of reference.[12] Neither Helmholtz nor Mach, however, established any kind of conceptual connection between the foundations of geometry and the relativity of motion – which, at the time, appeared to be entirely independent of one another. Yet it was an especially central feature of Kant's original approach to transcendental philosophy that the two were in fact closely connected. While Kant's answer to the question "how is pure mathematics possible?" essentially involved his distinctive perspective on Euclidean constructive operations, his answer to the question "how is pure natural science possible" involved an analogous constructive procedure by which Newton, from Kant's point of view, arrived at successive approximations to "absolute space" via a definite sequence of rule-governed operations starting with our parochial perspective here on earth and then proceeding to the center of mass of the solar system, the center of mass of the Milky Way galaxy, the center of mass of a system of such galaxies, and so on *ad infinitum*.[13] Indeed, the way in which Kant thereby established a connection between the problem of space and geometry and the problem of the relativity of motion was intimately connected, in turn, with both the overarching conception of the relationship between sensibility and understanding that frames his transcendental method and his characteristic perspective, more generally, on the relationship between constitutive and regulative transcendental principles.[14]

Now it was Mach, as I have suggested, who first forged a connection between Kant's original solution to the problem of "absolute space" and the late nineteenth-century solution based on the concept of an inertial frame of reference.[15] And it is clear,

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[12] For the nineteenth-century development of the concept of an inertial frame see DiSalle (1988, 1991); for Mach's place in this development see DiSalle (2002).

[13] Kant develops this interpretation of "absolute space" in his *Metaphysische Anfangsgründe der Naturwissenschaft* (1786), published between the first (1781) and second (1787) editions of the *Critique of Pure Reason*. For details see Friedman (1992, chapters 3 and 4), and also the Introduction to my (2004) translation of Kant's work.

[14] In particular, "absolute space," for Kant, is a regulative idea of reason, defined by the forever unreachable "center of gravity of all matter" which we can only successively approximate but never actually attain.

[15] Kant's construction of "absolute space," from a modern point of view, yields better and better approximations to a cosmic inertial frame of reference defined by the "center of gravity of all matter." Such a cosmic frame, in which the fixed starts are necessarily at rest, also counts as a surrogate for Newtonian "absolute space" in Mach's treatment: for details see again DiSalle (2002).

moreover, that Poincaré was familiar with this late nineteenth-century solution as well. However, it is also clear that Poincaré's attempt to base his discussion of the relativity of motion on his philosophy of geometry runs into serious difficulties at precisely this point; for Poincaré is here forced to distinguish his "law of relativity" from what he calls the "principle of relative motion." The latter applies only to inertial frames of reference, moving uniformly and rectilinearly with respect to one another, while the latter applies, as well, to non-inertial frames of reference in a state of uniform rotation: it follows from the "relativity and passivity" of space, for Poincaré, that uniform rotations of our coordinate axes should be just as irrelevant to the motions of a physical system as uniform translations. Therefore, the full "law of relativity," as Poincaré says, "ought to impose itself upon us with the same force" as does the more restricted "principle of relative motion." Poincaré must also admit, however, that the more extended "law of relativity" does not appear to be in accordance with our experiments (e.g., Newton's famous rotating bucket experiment).[16]

It is for this reason that Einstein's appeal to what he calls the "principle of relativity" in his 1905 paper on special relativity is entirely independent of Poincaré's "law of relativity," and it is also independent, accordingly, of Poincaré's "conventionalist" philosophy of geometry. Einstein's principle is limited, from the beginning, to inertial frames of reference (moving relative to one another with constant velocity and no rotation), and his concern is rather to apply this (limited) principle of relativity to both electro-magnetic and mechanical phenomena. Thus, in particular, whereas Poincaré's "law of relativity" involves very strong a priori motivations deriving from his philosophy of geometry (based on the "relativity and passivity of space"), Einstein's "principle of relativity" rests on the emerging experimental evidence suggesting that electro-magnetic and optical phenomena do not in fact distinguish one inertial frame from another. Einstein "conjectures" that this experimentally suggested law holds rigorously (and for all orders), and he proposes to "elevate" it to the status of a presupposition or postulate upon which a consistent electrodynamics of moving bodies may then be erected:

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[16] Poincaré formulates "the principle of relative motion" in chapter VII, "Relative Motion and Absolute Motion," of *La Science et l'Hypothèse* (1902, p. 135, 1913b, p. 107): "The motion of any system whatsoever must obey the same laws, whether it be referred to fixed axes, or to movable axes transported by a rectilinear and uniform motion. This is the principle of relative motion, which imposes itself upon us for two reasons: first, the most common experience confirms it, and second, the contrary hypothesis is singularly repugnant to the mind." This, of course, is the principle of what we now call Galilean relativity, which was originally formulated by Newton as Corollary V to the Laws of Motion, and then played a central role in the recent literature on inertial frames of reference (see the references cited in note 12 above). However, as Poincaré is well aware, such Galilean relativity holds only for (uniform) rectilinear motions and does not extend, therefore, to the case of (uniform) rotational motion Poincaré also wishes to subsume under his "law of relativity." Nevertheless, Poincaré says, "it seems that [the principle of relative motion] ought to impose itself upon us with the same force, if the motion is varied, or at least if it reduces to a uniform rotation" (1902, pp. 136–137, 1913b, p. 108). Thus, Poincaré's a priori commitment to the law of relativity, derived from the homogeneity and isotropy of space, stands in *prima facie* contradiction with the well-known experimental limitations of the principle of relative motion. (Poincaré presents a sophisticated analysis of this apparent contradiction in the following discussion, which I shall have to pass over here.)

> Examples of this sort [the relatively moving conductor and magnet—MF], together with the unsuccessful attempts to discover any motion of the earth relative to the "light medium," suggest that the phenomena of electrodynamics as well as mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics are valid. We will elevate [*erheben*] this conjecture (whose content will be called the "principle of relativity" in what follows) to the status of a postulate [*Voraussetzung*], and also introduce another postulate, which is only apparently irreconcilable with it, namely, that light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body. These two postulates suffice for attaining a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. (1905, pp. 891–892, 1923, pp. 37–38)

Hence, Einstein's understanding of the principle of relativity is also entirely independent of Poincaré's carefully constructed hierarchy of the mathematical sciences, and it is for precisely this reason, I suggest, that Poincaré himself could never accept Einstein's theory.[17]

Nevertheless, it appears overwhelmingly likely that, although Einstein did not embrace Poincaré's "conventionalist" philosophy of geometry, Einstein's use of the principle of relativity was explicitly inspired by Poincaré's more general methodology described in *La Science et l'Hypothèse* – according to which the fundamental principles of mechanics, in particular, are "conventions or definitions in disguise" arising from "experimental laws" that "have been elevated into principles to which our mind attributes an absolute value."[18] In Einstein's case, the experimental law in question comprises the

[17] In his 1912 lecture on "Space and Geometry," appearing in Poincaré (1913a), Poincaré explicitly considers what we now call the four-dimensional geometry of Minkowski space–time, and he clearly states his preference for an alternative formulation of the Lorentzian type – where, in particular, both the Newtonian laws of mechanics and "the relativity and passivity of space" retain a foundational role. Thus, from a modern point of view, while Poincaré's most fundamental "law of relativity" is a purely geometrical principle, expressing the necessary symmetries of three-dimensional (homogeneous) space, Einstein's "principle of relativity" expresses the symmetry or invariance properties of the laws of Maxwell–Lorentz electrodynamics – which we now take to be the symmetries of Minkowski space–time. The central problem with Poincaré's hierarchy, from this point of view, is that it makes the three-dimensional geometry of space prior to the four-dimensional geometry of space–time: compare again DiSalle (2006, section 3.7) for a similar diagnosis.

[18] This idea is stated as a key part of Poincaré's "General Conclusions" to his discussion of (classical) mechanics (1902, p. 165, 1913b, p. 125): "[The fundamental principles of mechanics] are conventions or definitions in disguise. Yet they are drawn from experimental laws; these laws, so to speak, have been elevated [*érigées*] into principles to which our mind attributes an absolute value." Later, in *Geometrie und Erfahrung* (1921), Einstein explicitly uses the language of "elevation" [*erheben*] in connection with precisely Poincaré's "conventionalism" (1921, p. 8, 1923, p. 35): "Geometry (*G*) [according to Poincaré's standpoint] asserts nothing about the behavior of actual things, but only geometry together with the totality (*P*) of physical laws. We can say, symbolically, that only the sum (*G*) + (*P*) is subject to the control of experience. So (*G*) can be chosen arbitrarily, and also parts of (*P*); all of these laws are conventions. In order to avoid contradictions it is only necessary to choose the remainder of (*P*) in such a way that (*G*) and the total (*P*) together do justice to experience. On this conception axiomatic geometry and the part of the laws of nature that have been elevated [*erhobene*] to conventions appear as epistemologically of equal status." (I shall return to *Geometrie und Erfahrung* below.) To the best of my knowledge, this striking language in Einstein's 1905 paper (together with its reappearance in 1921) has not been previously noted in the literature.

recent results in electrodynamics and optics, and Einstein now proposes to "elevate" both the principle of relativity and the light principle (which together imply that the velocity of light is invariant in all inertial frames) to the status of "presuppositions" or "postulates." These two postulates together then allow us to "stipulate" a new "definition of simultaneity" (based on the assumed invariance of the velocity of light) implying a radical revision of the classical kinematics of space, time, and motion. In particular, whereas the fundamental kinematical structure of an inertial frame of reference, in classical mechanics, is defined by the Newtonian laws of motion, (a revised version of) this same structure, in Einstein's theory, is rather defined by his two postulates.[19]

A central contention of Kant's original version of transcendental philosophy, as we know, is that the Newtonian laws of motion are not mere empirical laws but a priori constitutive principles on the basis of which alone the Newtonian concepts of space, time, and motion can then have empirical application and meaning. What we have just seen is that Einstein's two fundamental "presuppositions" or "postulates" play a precisely parallel role in the context of special relativity. But we have also seen significantly more. For Poincaré's conception of how a mere empirical law can be "elevated" to the status of a "convention or definition in disguise" is a continuation, in turn, of Kant's original conception of the constitutive a priori. Whereas Helmholtz's principle of free mobility generalized and extended Kant's original theory of geometrical construction within our "subjective" and "*necessary* form of external intuition," Poincaré's idea that specifically Euclidean geometry is then imposed on this form by a "convention or definition in disguise" represents an extension or continuation of Helmholtz's conception. In particular, specifically Euclidean geometry is applied to our experience by precisely such a process of "elevation," in which the merely empirical fact that this geometry governs, very roughly and approximately, our actual perceptual experience of bodily displacements gives rise to a precise mathematical framework within which alone our properly physical theories can subsequently be formulated.[20]

This same process of "elevation," in Einstein's hands, then makes it clear how an extension or continuation of Kant's original conception can also accommodate new and surprising empirical facts – in this case, the very surprising empirical discovery (to one

[19] The crucial point, in this connection, is that Newton's third law – the equality of action and reaction – implicitly defines the relation of absolute simultaneity in a classical inertial frame, in so far as it allows us to coordinate action-reaction pairs related by the Newtonian law of (instantaneously propagated) gravitation. Einstein's two postulates take over precisely this role in the case of his new, relativized relation of simultaneity defined by (continuously propagated) electro-magnetic processes.

[20] Euclidean geometry is singled out, for Poincaré, in that it is both mathematically simplest and very naturally corresponds – roughly and approximately – to our pre-scientific experience of bodily displacements. Just as Helmholtz's conception, as I have suggested, is the minimal extension of Kant's original conception consistent with the discovery of non-Euclidean geometries (compare note 8 above), Poincaré's conception is the minimal extension of Helmholtz's consistent with the more sophisticated group-theoretic version of the principle of free mobility due to Sophus Lie, the new perspective on the relativity of motion due to the modern concept of an inertial frame, and, most importantly, the apparently paradoxical new situation in electrodynamics arising in connection with precisely this relativity of motion – where, in particular, Poincaré's hierarchical conception of the mathematical sciences allows him to retain the foundational role of both Euclidean spatial geometry and the laws of Newtonian mechanics in the face of what we now call Lorentzian (as opposed to Galilean) relativity (compare note 17 above).

or another degree of approximation) that light has the same constant velocity in every inertial frame. It now turns out, in particular, that we can not only impose already familiar and accepted mathematical frameworks (Euclidean geometry) on our rough and approximate perceptual experience, but, in appropriate circumstances, we can also impose entirely unfamiliar ones (the kinematical framework of special relativity). Einstein's creation of special relativity, from this point of view, thus represents the very first instantiation of a relativized and dynamical conception of the a priori – which, in virtue of precisely its historical origins, has a legitimate claim to be considered as genuinely constitutive in the transcendental sense. And what vindicates this claim, accordingly, is a reconceived version of transcendental philosophy where precisely the kind of integrated intellectual history I have been trying to exemplify takes the place of Kant's original transcendental faculty psychology: in particular, that the "inner logic" of the successive intellectual situations in question proceeds against the background of, and explicitly in light of, Kant's original theory is what makes this enterprise properly "transcendental."

Yet Einstein's creation of the general theory of relativity in 1915 involved an even more striking engagement with Poincaré's "conventionalist" methodology, which, I contend, makes the transcendentally constitutive role of this theory's fundamental postulates (the light principle and the principle of equivalence) even more evident.

The first point to make, in this connection, is that the principle of equivalence (together with the light principle) plays the same role in the context of the general theory that Einstein's two fundamental "presuppositions" or "postulates" played in the context of the special theory: namely, they define a new inertial-kinematical structure for describing space, time, and motion. Because Newtonian gravitation theory involves an instantaneous action at a distance (and therefore absolute simultaneity: compare note 18 above), it was necessary after special relativity to develop a new theory of gravitation where the interactions in question propagate with the velocity of light. And Einstein solved this problem, via the principle of equivalence, by defining a new inertial-kinematical structure wherein the freely falling trajectories in a gravitational field replace the inertial trajectories described by free particles affected by no forces at all. The principle of equivalence, in this sense, replaces the classical law of inertia holding in both Newtonian mechanics and special relativity. But the principle of equivalence itself rests on a well-known empirical fact: that gravitational and inertial mass are equal, so that all bodies, regardless of their mass, fall with exactly the same acceleration in a gravitational field. In using the principle of equivalence to define a new inertial-kinematical structure, therefore, Einstein has "elevated" this merely empirical fact (recently verified to a quite high degree of approximation by Lorand von Eötvös) to the status of a "convention or definition in disguise" – just as he had earlier undertaken a parallel "elevation" in the case of the new concept of simultaneity introduced by the special theory.[21]

[21] Friedman (2001, pp. 86–91) develops more fully the parallel between these two cases of "elevating" a mere empirical fact to the status of a (relativized) a priori principle by first examining the relationship between the invariance of the velocity of light (as recently verified in the Michelson–Morley experiment) and Einstein's new definition of simultaneity, and then the relationship between the equality of gravitational and inertial mass (as recently verified in the Eötvös experiments) and the principle of equivalence.

Nevertheless, Einstein did not reach this understanding of the principle of equivalence all at once. He first operated, instead, within an essentially three-dimensional understanding of special relativity, and he proceeded to develop relativistically acceptable models of the gravitational field by considering the inertial forces (like centrifugal and Coriolis forces) arising in non-inertial frames of reference within this framework.[22] It was in precisely this context, in particular, that Einstein came upon the example of the uniformly rotating frame (the rotating disk), and it is at this point (and only at this point) that he then arrived at the conclusion that the gravitational field may be represented by a non-Euclidean geometry. This use of non-Euclidean geometry, however, was essentially three-dimensional, limited to purely *spatial* geometry, and Einstein did not arrive at the idea of a four-dimensional non-Euclidean geometry – where *space-time* geodesics represent freely falling trajectories affected only by gravitation – until he had generalized his conception to what we now call the four-dimensional (semi-)Riemannian geometries of variable curvature.[23]

It was in precisely the context of this line of thought, finally, that Einstein found that he now had explicitly to oppose Poincaré's "conventionalist" philosophy of geometry. Yet Einstein's argument – as described in *Geometrie und Erfahrung* (1921) – was far from a simple rejection of Poincaré's methodology in favor of a straightforward "empiricism."[24] For Einstein also famously says, in the same work, that "*sub specie aeterni*" Poincaré is actually correct – so that, in particular, Einstein's reliance on a Helmholtzian conception of "practically rigid bodies" is here merely provisional. I have suggested, therefore, that we can best understand Einstein's procedure as one of delicately situating himself *between* Helmholtz and Poincaré. Whereas Einstein had earlier followed Poincaré's general "conventionalist" methodology in "elevating" the principle of relativity (together with the light principle) to the status of a "presupposition" or "postulate," he here follows Helmholtz's "empiricism" in rejecting Poincaré's more specific philosophy of geometry in favor of "practically rigid bodies."[25]

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[22] See Norton (1985) (1989) for the details of Einstein's early applications of the principle of equivalence.

[23] I discuss at length the crucial importance of the rotating disk example in the development of Einstein's thought – following Stachel (1980) (1989) – in Friedman (2001, 2002).

[24] Compare note 18 above; and again, for a detailed analysis of *Geometrie und Erfahrung*, against the background of both Helmholtz and Poincaré, see Friedman (2001, 2002).

[25] Einstein does not explicitly mention Helmholtz in *Geometrie und Erfahrung*. However, in a closely related article on "Non-Euclidean Geometry and Physics," Einstein makes it perfectly clear that the opposition he has in mind is precisely that between Helmholtz and Poincaré (1925, pp. 18–19): "Either one accepts that the 'body' of geometry is realized in principle by the solid bodies of nature, if only certain prescriptions are maintained regarding temperature, mechanical stress, and so on; this is the standpoint of the practicing physicist. Then a natural object corresponds to the 'interval' of geometry, and all propositions of geometry thereby attain the character of assertions about real bodies. This standpoint was represented especially clearly by Helmholtz, and one can add that without it establishing the [general—MF] theory of relativity would have been practically impossible. Or, one denies in principle the existence of objects that correspond to the fundamental concepts of geometry. Then geometry alone contains no assertions about objects of reality, but only geometry together with physics. This standpoint, which may be more perfect for the systematic

It does not follow, however, that Einstein is also rejecting his earlier embrace of Poincaré's general "conventionalist" methodology. Indeed, Einstein had already side-stepped Poincaré's philosophy of geometry in the case of special relativity, and for essentially the same reason he explicitly opposes it here: Poincaré's rigid hierarchy of the sciences, in both cases, stands in the way of the radical new innovations Einstein himself proposes to introduce.[26]

But why was it necessary, after all, for Einstein to engage in this delicate dance between Helmholtz and Poincaré? The crucial point is that Einstein thereby arrived at a radically new conception of the relationship between the foundations of (physical) geometry and the relativity of space and motion. These two problems, as we have seen, were closely connected in Kant, but they then split apart and were pursued independently in Helmholtz and Mach (compare the paragraph to which note 11 above is appended). In Poincaré, as we have also seen, the two were perceptively reconnected once again, in so far as Poincaré's hierarchical conception of the mathematical sciences incorporated both a modification of Helmholtz's philosophy of geometry and a serious engagement with the late nineteenth-century concept of inertial frame (compare note 15 above, together with the paragraph to which it is appended, and also note 19 above). Indeed, it is for precisely this reason, as we now see, that Poincaré's scientific epistemology was so important to Einstein. Einstein could not simply rest content with Helmholtz's "empiricist" conception of geometry, because the most important problem with which he was now faced was to connect the foundations of geometry with the relativity of motion. But Einstein could not rest content with Poincaré's conception either, because his new models of gravitation had suggested that geometry has genuine physical content.

Einstein's radically new way of reconfiguring the relationship between the foundations of geometry and the relativity of motion therefore represents a natural (but also entirely unexpected) extension or continuation of the same conception of dynamical and relativized constitutive a priori principles he had first instantiated in the creation of special relativity (compare note 20 above, together with the paragraph to which it is appended and the following paragraph). Just as he had

presentation of a completed physics, was represented especially clearly by Poincaré. On this standpoint the total content of geometry is conventional; which geometry is to be preferred depends on how 'simple' a physics can, by its use, be established in agreement with experience." Ryckman (2005, section 3.3) emphasizes the importance of this passage in relation to the earlier argument of *Geometrie und Erfahrung*.

[26] See note 17 above, together with the paragraph to which it is appended. As I suggested, Einstein could not embrace Poincaré's philosophy of geometry even in 1905, since it privileges a priori the three-dimensional geometry of space over the *de facto* symmetries of the laws of motion (which, on our current understanding, express the four-dimensional geometrical symmetries of space–time). Einstein's divergence from Poincaré on this point is even stronger in general relativity; for, not only do we now use non-Euclidean geometries to describe both space and space–time, but we have also definitively given up (in both cases) the homogeneity and isotropy (constant curvature) of the underlying geometry. Einstein thereby ultimately arrived at a radically new conception of physical geometry envisioned by neither Helmholtz nor Poincaré (compare again note 8 above): for details see again Friedman (2002).

earlier shown how an extension or continuation of Kant's original conception could accommodate new and surprising empirical facts (the discovery of the invariance of the velocity of light), Einstein here shows how a further extension of this same tradition can do something very similar in facilitating, for the first time, the application of a non-Euclidean geometry to nature. In this case, however, it is not the relevant empirical fact (the well-known equality of gravitational and inertial mass) that is surprising, but the entirely unforeseen connection between this fact and the new geometry. And what makes this connection itself possible, for Einstein, is precisely the principle of equivalence – which thereby constitutively frames the resulting physical space-time geometry of general relativity in just the same sense that Einstein's two fundamental "presuppositions" or "postulates" had earlier constitutively framed his mathematical description of the electrodynamics of moving bodies in special relativity (within what we now call the geometry of Minkowski space–time). Whereas the particular geometry in a given general relativistic space–time is now determined entirely empirically (by the distribution of mass and energy in accordance with Einstein's field equation), the principle of equivalence itself is not empirical in this sense. This principle is instead *presupposed* – as a transcendentally constitutive condition – for any such geometrical description of space–time to have genuine empirical meaning in the first place.

The historicized version of transcendental philosophy I am attempting to exemplify therefore sheds striking new light, I believe, on the truly remarkable depth and fruitfulness of Kant's original version. Kant's particular way of establishing a connection between the foundations of geometry and the relativity of motion – which, as we have seen, lies at the heart of his transcendental method (compare the paragraph to which note 6 above is appended, together with the following paragraph) – has not only lead, through the intervening philosophical and scientific work of Helmholtz, Mach, and Poincaré, to a new conception of the relativized a priori first instantiated in Einstein's theories, it has also led, through this same tradition, to a radically new reconfiguration of the connection between geometry and physics in the general theory of relativity itself. There can be no question, of course, of Kant having "anticipated" this theory in any way. The point, rather, is that Kant's own conception of the relationship between geometry and physics (which was limited, of necessity, to Euclidean geometry and Newtonian physics) then set in motion a remarkable series of successive reconceptualizations of this relationship (in light of profound discoveries in both pure mathematics and the empirical basis of mathematical physics) that finally eventuated in Einstein's theory.

References

DiSalle, R. (1988). *Space, Time, and Inertia in the Foundations of Newtonian Physics*. Doctoral Dissertation. Chicago, IL: University of Chicago.

DiSalle, R. (1991). "Conventionalism and the Origins of the Inertial Frame Concept." *PSA 1990* 2: 139–147.

DiSalle, R. (2002). "Reconsidering Ernst Mach on Space, Time, and Motion." In D. Malament, ed. *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*. Chicago, IL: Open Court, pp. 167–191.

DiSalle, R. (2006). *Understanding Space-Time: The Philosophical Development of Physics from Newton to Einstein*. Cambridge: Cambridge University Press.

Einstein, A. (1905). "Zur Elektrodynamik bewegter Körper." *Annalen der Physik* 17: 891–921. Translated by G. Jeffrey and W. Perrett (1923) as *On the Electrodynamics of Moving Bodies* in Lorentz et al. *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theories of Relativity*. London: Methuen.

Einstein, A. (1921). *Geometrie und Erfahrung. Erweiterte Fassung des Festvortrages gehalten an der Preussischen Akademie der Wissenschaft zu Berlin am 27. Januar 1921*. Berlin: Springer. Translated by G. Jeffrey and W. Perrett (1923) as *Geometry and Experience* in *Sidelights on Relativity*. London: Methuen.

Einstein, A. (1925). "Nichteuklidische Geometrie und Physik." *Die neue Rundschau* 36: 16–20.

Folina, J. (1992). *Poincaré and the Philosophy of Mathematics*. New York: Macmillan.

Friedman, M. (1992). *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

Friedman, M. (1997). "Helmholtz's *Zeichentheorie* and Schlick's *Allgemeine Erkenntnislehre*: Early Logical Empiricism and Its Nineteenth-Century Background." *Philosophical Topics* 25: 19–50.

Friedman, M. (1999). *Reconsidering Logical Positivism*. Cambridge: Cambridge University Press.

Friedman, M. (2000). "Geometry, Construction, and Intuition in Kant and His Successors." In G. Scher and R. Tieszen, eds. *Between Logic and Intuition: Essays in Honor of Charles Parsons*. Cambridge: Cambridge University Press.

Friedman, M. (2001). *Dynamics of Reason. The 1999 Kant Lectures at Stanford University*. Stanford: CSLI Publications.

Friedman, M. (2002). "Geometry as a Branch of Physics: Background and Context for Einstein's 'Geometry and Experience.' " In D. Malament, ed. *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*. Chicago, IL: Open Court, pp. 193–229.

Friedman, M., trans. & ed. (2004). *Immanuel Kant: Metaphysical Foundations of Natural Science*. Cambridge: Cambridge University Press.

Hatfield, G. (1990). *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*. Cambridge, MA: MIT Press.

Norton, J. (1985). "What Was Einstein's Principle of Equivalence?" *Studies in History and Philosophy of Science* 16: 203–246. Reprinted in D. Howard and J. Stachel, eds. (1989). *Einstein and the History of General Relativity*. Boston, MA: Birkhäuser.

Poincaré, H. (1902). *La Science et l'Hypothèse*. Paris: Flammarion. Translated as *Science and Hypothesis* in Poincaré (1913b).

Poincaré, H. (1913a). *Dernières Pensées*. Paris: Flammarion. Translated by J. Buldoc (1963) as *Mathematics and Science: Last Essays*. New York: Dover.

Poincaré, H. (1913b). *The Foundations of Science*. Translated and edited by G. Halstead. Lancaster: Science Press.

Reichenbach, H. (1920). *Relativitätstheorie und Erkenntnis Apriori*. Berlin: Springer. Translated by M. Reichenbach (1965) as *The Theory of Relativity and A Priori Knowledge*. Berkeley/Los Angeles, CA: University of California Press.

Ryckman, T. A. (2005). *The Reign of Relativity: Philosophy in Physics 1915–1925*. Oxford: Oxford University Press.

Stachel, J. (1980). "Einstein and the Rigidly Rotating Disk." In A. Held, ed. *General Relativity and Gravitation*. New York: Plenum. Reprinted as "The Rigidly Rotating Disk as the 'Missing Link' in the History of General Relativity" in D. Howard and J. Stachel, eds. (1989). *Einstein and the History of General Relativity*. Boston, MA: Birkhäuser.