How do we *explain* mathematical explanation of scientific facts?

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During the last years the discussion concerning *mathematical explanation* has become of much importance in the philosophy of mathematics. The cause of this interest might be traced back in a recent revival in the analytic literature around the topic, and in the impossibility to give an account of mathematical explanation starting from general theories of scientific explanation. The topic of mathematical explanation has, as we’ll see, great importance not only in the philosophy of mathematics, but also in different areas including epistemology, metaphysics, philosophy of physics and history and philosophy of science in general.

What do we mean with the term “mathematical explanation”? It could be understood in two very different senses: in general we can speak of *mathematical explanation of mathematical facts*, and *mathematical explanation of scientifical facts*. The first sense is “confined” to itself and concerns the universe of informal and formal proofs within mathematics. Here the questions are: “Does this proof convince\(^1\)? And does it make *why* the result more intelligible\(^2\)? It’s the question posed by Lakatos in his famous informal proof of Euler’s formula on simple polyedra \(V - E + F = 2\): “What does a mathematical proof prove?” [Lakatos, 1976] [Lakatos, 1978]\(^3\). This first sense of explanation has obviously a richer story than the second (or, more probably, they have both a rich story but the second story hasn’t been written

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\(^1\)Conviction of the validity of a proof is tied to mathematical practice. See, for example, the problem of manipulation of non-homogeneous quantities before Descartes. We have some “departures” from this euclidean tradition in the Arab world, with the algebra of Abu Kamil (Xth century) or Al’Samaw’al (XIIth century), but in general the euclidean framework did make not possible to demonstrate theorems in algebra or geometry manipulating non-homogeneous quantities until the XVIIth century.

\(^2\)In the sense of the definition given in page 3.

\(^3\)For examples of informal proofs in mathematics see also Polya [Polya, 1954]
yet⁴). As stressed in the title, we’re going to deal here with mathematical explanation in the second sense. So, from now on I’ll speak of mathematical explanation referring to mathematical explanation of scientific facts, and in particular to the explanatory role of mathematics in physical phenomena. I’m going to illustrate this second sense of explanation with an example (it’s Feynman’s storyboard with my dialogues):

A child asks his father: “Daddy, why that apple has fallen to earth?”.

The father does not have any good answer. (“Why not?”, he seems to think).

The child becomes a young boy, and he asks his girlfriend: “Why the phenomenon of tides?”. No reason. When the boy finally comes to college he asks his physics teacher why the planets turn around the sun. “Because of the Gravitational Force”, the teacher replies. “What’s that?”. So the teacher writes on the blackboard the famous formula:

\[ F = \frac{G m_i m_j}{r_{ij}^2} \]  

(1)

Feynman remarks that both Newton’s law, the principle of minimum action and the method of local field are three mathematical formulations of gravitational law which give exactly the same mathematical result [Feynman, 1967, p. 47]. But they are all, like the description of the teacher in the example, mathematical descriptions of a fundamental law. So, it seems here that we need (and we have) only mathematics in order to explain what the phenomenon of gravitation is.

After having “explained” the phenomenon (after having given a possible reason to the question “Why this phenomenon?”) via the use of a mathematical structure⁵ (or a model, in a very naïf sense), we are left with some more general questions about the applicability of mathematics and the ontology of mathematical objects. This is another important point we want to underline: the topic of explanation is deeply connected to the different topics of application of mathematics to natural sciences, mathematization and ontological questions which are well-expressed by the famous indispensability argument. How does mathematics hooks on reality? The fact that we are not able to split explanation from this question is well expressed by Shapiro:

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⁴The problem of explanation is different from the problem of application. Application problem’s story starts from Plato. But, if we consider explanation as a sub-argument of application, we might say that explanation has the same story of application, also if this story hasn’t been written yet. This is my point here.

⁵Here I use the term mathematical structure in a very general sense. Any suggestion?
Strictly speaking, a mathematical description, model, structure, theory, or whatever, cannot serve as an explanation of a non-mathematical event without an account of the relationship between mathematics per se and scientific reality per se. Without such an account, it is not clear how “scientific explanations” succeed in explaining anything. That is, one cannot begin to account for how science contributes to knowledge without an account of what mathematical-scientific activity has to do with the reality of which science contributes knowledge. This problem becomes particularly important in the context of philosophy of science and philosophy of mathematics [Shapiro, 1983, p. 525].

M. Duhem could have answered back to Shapiro:

Une théorie physique sera donc un système de propositions logiquement enchaînées, et non pas une suite incohérente de modèles mécaniques ou algébriques; ce système n’aura pas pour object de fournir une explication, mais une représentation et une classification naturelle d’un ensemble de lois expérimentales [Duhem, 1906, p. 157].

Giving an account of “what mathematical-scientific activity has to do with the reality of which science contributes knowledge” is very far from our will (and from my possibilities...). But Duhem’s position towards the “unreasonable effectiveness of mathematics in the natural sciences” [Wigner, 1967] seems to be more “magical” and mysterious than Shapiro’s reflections. As Mancosu has pointed out, today nobody has really an account of mathematical explanations of scientific phenomena [Hafner and Mancosu, 2005]. We don’t want to discuss here ontological questions, but from our example 1 (and from our everyday life) it’s evident that mathematical structures are necessary in order to understand, and sometimes explain, the phenomena. In particular, if we follow the definition of “The Concise Oxford Dictionary”, the verb “to explain” means: 1) “Make clear or intelligible with detailed information”, 2) “Account for”, 3) “Minimize the significance of (a difficulty or a mistake) by explanation”. The verb is borrowed from the classical latin verb “explanare”, which means “to make plain, to flatten” (ex -out-, planus -plain-). So, in this sense, our formulation of the law of Gravitation stands as an explanation to the question “Why?”.

But, even if we are not able to give a precise account of a mathematical explanation (explanation toujours in the sense 2), we might ask ourself: “When do

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6here we’re using the verb ”to explain” in a very general and inaccurate sense. Our aim is to clarify the sense of this verb in this context.
we have a mathematical explanation of physical phenomena?"; from which
follows the other question: “Is there something common to those explana-
tions?”. I’ll present here only one general position on explanation, and I’ll
sketch some main lines connected to this topic.
But, before doing that, I want to come back to the difference between appli-
cation of mathematics to natural sciences and mathematization:

We should speak of application of mathematics to a natural or
social science when a natural phenomenon that such a science is
studying is described or explained using mathematics, or when a
problem this science aims to solve is answered by means of math-
ematical techniques. By contrast, we should speak of mathematiza-
tion only when the object of this science becomes a mathemat-
ical object, i. e. mathematics provides a model or a scheme of a
natural or social phenomenon and this model or scheme becomes
the real object of studying [Panza, 2002, p. 253-254].

This distinction is very useful in order to analyze the role of mathe-
matical structures in natural phenomena through history and look at the
sub-argument of mathematical explanation in both the notions. Those two
notions let us consider models, and the role of mathematical and physical
analogy in models [Israel, 1996].

A first tentative of giving a definition of mathematical explanation of
physical phenomena was made by Mark Steiner [Steiner, 1978b]. We’ll sketch
here is main idea, which is deeply connected with his idea of mathematical
explanation of mathematical facts [Steiner, 1978a]. Steiner starts asking him-
self two different questions:

1. Do physical phenomena have mathematical explanations?

2. If so, what existential conclusions follows? Do such explanations make
   reasonable the existence of mathematical entities?

He discusses a single example: the motion of a rigid body about a fixed point.
This motion can always be achieved by rotating the body of a certain angle
about a fixed axis. The result comes from Euler’s theorem for rigid body
motion:

The general displacement of a rigid body with one point fixed is
a rotation about some axis [Goldstein, p. 156].

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7 I claim that with modern XXth’s physics explanation has become a sub-argument also
of mathematization. We could see an example of this in the story of positrons, which were
first postulated in 1928 by Paul Dirac as a consequence of the Dirac equation.
If the matrix $A$ describing the motion of the body is a rotation, as stated by the theorem, it’s characteristic of a rotation that one direction (the axis of the rotation) is left unaffected by the operation and that the magnitude of the vectors be unaffected by the trasformation (this last condition is provided by the orthogonality conditions). From Euler’s theorem follow some important results for a proper rotation $A$: (a) the product of any eigenvalue with its complex conjugate is 1. (b) the complex conjugate of an eigenvalue is an eigenvalue. (c) the product of all the eigenvalues is 1. Putting together the conditions we have that one of the eigenvalues must be +1; so, there is a real vector $x$ such that $Ax = x$; and this vector $x$ is an axis of rotation.

Steiner claims this is a mathematical explanation of a physical fact, though physical assumptions enter: that physical space is three-dimensional. His point is that we have a mathematical explanation of a physical fact when we remove the physics and we are left with a mathematical explanation of a mathematical facts. In his words:

I shall not reproduce my analysis of mathematical explanation here, but assume that mathematical explanation of mathematical truth exists. The difference between mathematical and physical explanations of physical phenomena is now amenable to analysis. In the former, as in the latter, physical and mathematical truths operate. But only in mathematical explanation is this the case: when we remove the physics, we remain with a mathematical explanation—of a mathematical truth! In our example, the “bridge” between physics and mathematics is the assumptions that space is three-dimensional Euclidean, and that the rotation of a rigid body around a point generates an orthogonal, real, proper transformation (to use the lingo). Deleting these assumptions, we obtain an explanatory proof of a theorem concerning transformations and eigenvectors. In standard scientific explanations, after deleting the physics nothing remains [Steiner, 1978b, p. 19].

His reason to the first question “Do physical phenomena have mathematical explanations?” is then positive. Steiner’s reference to “mathematical truths” introduces us to his second (ontological) question “Do such explanations make reasonable the existence of mathematical entities?”. We are not going to discuss this point here but we note that, by contrast, Steiner’s reply to the latter is negative. For him mathematical explanation of physical phenomena could not be used to infer the existence of mathematical entities, for the existence of mathematical entities is presupposed in the description.
of the phenomena to be explained (explananda). The argument of Mark Steiner is very important in order to try to give a possible reason to the question “When do we have a mathematical explanation of physical phenomena?”.

Steiner’s discussion is also important because, focusing on ontological questions, underlines a topic which is strictly connected with mathematical explanation of physical phenomena: how can a nominalist account for the explanatoriness of mathematics in the empirical sciences? What is a good nominalistic reason to the indispensability of mathematical entities involved in this sense of explanatoriness?

A position very different from Steiner’s is Baker’s [Baker, 2005]. His argument, which concerns a case-study from evolutionary biology, is based on a new line of the indispensability argument which does not presuppose holism. For Baker there exist what he calls “genuine mathematical explanations of physical phenomena”, and the explanation of the prime cycle length (13 and 17 years) of the life-cycle of periodical cicadas using number theory is an example of this. Very roughly, the argument is the following: Why are the life-cycle periods of cicada primes? There are two biological theories which share number theoretical bases. The number theoretic theorem “prime periods minimize intersection (compared to non-prime periods)” is essential to the structure of the general explanation (which makes also use of specific ecological facts and general biological laws) and answers to the particular question: “Why the prime periods are evolutionarily advantageous?”. Then Baker (as Steiner) comes to the problem of ontology. Here the question is: Is the cicada example a genuinely mathematical explanation? The case-study of cicada is useful to the platonist only if: a) the application is external to mathematics, b) the phenomenon must be in need of explanation, c) the phenomenon must have been identified independently of the putative explanation. The three conditions are satisfied by the example, so we need to check if the mathematical component of the explanation is explanatory in its own right. A casual-account is problematic in order to doing this, but if we adopt the deductive-nomological model (D-N model) or a pragmatic account, we can assert that the mathematical component is explanatory in its own right. In the conclusion Baker points out that:

... there are genuine mathematical explanations of physical phenomena, and that the explanation of the prime cycle lengths of periodical cicadas using number theory is one example of such. If

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8 this is very different, as Steiner points out, from the presupposition of light quanta in the photoelectric effect.

9 For details see [Baker, 2005, p. 234-235]
this is right, then applying inference to the best explanation in the cicada example yields the conclusion that number exist [Baker, 2005, p. 236].

An original approach to mathematical explanation in physics comes from Batterman’s definition of asymptotic reasoning [Batterman, 2001], a kind of abstract reasoning which involves methods named asymptotic methods. Those methods represent for Batterman the way in which science often proceeds: eliminating details and, in some sense, precision; the details of a system are, for the most part, unnecessary to the comprehension of the his behavior. In this sense, asymptotic methods play an explanatory and interpretative role, especially when we take into account properties which emerge in the asymptotic domains between two theories (a sort of no man’s land between two theories). They might play an important role in the philosophical investigation of intra-theoretical domains and contexts of reduction.

I’ve sketched the main idea of Mark Steiner and Alan Baker in order to make clear that the sense of mathematical explanation we’re dealing with could be very different from author to author and involves more general problematics. I want to conclude this short exposition emphasizing that since now we have not spoken of the role of mathematical practice and history in such a debate. New positions in the philosophy of mathematics take into account informal arguments and the grow of mathematical knowledge, and try to benefit from the interplay between history and philosophy of mathematics [Lakatos, 1978][Tymoczko, 1998][Kitcher, 1984]. As new and interesting philosophical reflections are emerging around mathematical explanations of physical phenomena, the role of mathematical practice and the development of the concept of explanation through the history of application of mathematics and of mathematization could give new contributes to the discussion.

A very good example of this interplay comes from the book of Giorgio Israel on models [Israel, 1996]. His study of modelization in natural and social sciences, and his rational description of the “oscillatory” passage from “mathematical analogies” to “physical analogies” in the history of mathematical modelization (“l’histoire de la modélisation mathématique à toujours oscillé - et continue d’osciller - entre le Scylla de l’analogie mathématique et le Charybde de l’analogie mécanique et de l’approche mécaniste” [Israel, 1996, p. 82]), might be a good starting point in order to focus on:

• the role of analogies (physicals and mathematicals) in the context of explanation of a physical phenomenon -and this could help us in an-
answering the question “When do we have mathematical explanations of non-mathematical events?”.

- an approach to the problem in which a particular explanation is associated to a particular mathematical description of a particular non-mathematical event—as happens for models.

The latter approach, renouncing to a unitary mathematical explanation of the reality and then to a uniform solution, might be compared to different positions which appeals to a more comprehensive view such as Kitcher’s unification model [Kitcher, 1989].

The debate on mathematical explanation of physical facts is so open and represents a very important and basic topic in the philosophy of mathematics. It is a thematic which might take profit from different and very distant areas of knowledge.

If we substitute “Description of reality” for “Literature” and “mathematical explanations” for “language” in the following quote from Roland Barthes, we are optimist in thinking that the sense of explanation we’ve been talking about will give work to philosophers of science for some time yet:

La multiplication des écritures institue une Littérature nouvelle dans la mesure où celle-ci n’invente son langage que pour être un projet: la Littérature devient l’utopie du langage [Barthes, 1953, p. 76].

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References


